Abstract—We propose and experimentally demonstrate a Tomlinson–Harashima precoding (THP) scheme with the coefficients computed by simulations. It realizes spectrally efficient faster-than-Nyquist (FTN) signaling for quadrature amplitude modulation (QAM) format. In our scheme, the coefficients of the THP precoder are obtained by numerical simulations with an additive white Gaussian noise (AWGN) channel, in which the intersymbol interference (ISI) is induced by a digital FTN filter. Then, we experimentally apply these THP coefficients at the transmitter to mitigate the FTN-related ISI, while the ISI from the analog channel is suppressed by the receiver-side adaptive equalizer. By this means, the experimental estimation of the THP coefficients is not required, leading to a simplified system operation. We perform numerical simulations to study the impacts of FTN signaling and channel bandwidth limitation on the proposed THP scheme. A proof-of-concept experiment is conducted, in which a 28-Gbaud FTN single-sideband (SSB) 16-QAM signal is transmitted over an 80-km single mode fiber (SMF). Compared to the conventional scheme with the experimentally estimated THP coefficients, a negligible sensitivity penalty is observed for the proposed scheme. We also achieve a record FTN rate of 21.73% and a high spectral efficiency of 4.30 b/s/Hz for the single-polarization 16-QAM format on a single wavelength.

Index Terms—Direct detection, faster-than-Nyquist, Tomlinson–Harashima precoding.

I. INTRODUCTION

In data center interconnects (DCI) and metro applications, direct detection (DD) system is a cost-effective solution. Compared to a conventional intensity modulation DD (IM-DD) scheme [1], [2], a single-sideband self-coherent detection (SSB SCD) system enables an increased capacity and a longer transmission distance [3]–[5]. By employing high-order quadrature amplitude modulation (QAM) format, the SSB SCD system can approximate the same optical spectral efficiency as a single-polarization coherent system. To further improve the optical spectral efficiency of a DD system, faster-than-Nyquist (FTN) signaling is an attractive approach by increasing the symbol rate above the bandwidth limit, characterized as the FTN rate of (Nyquist bandwidth / system bandwidth – 1) [6]. The Nyquist bandwidth equals half of the symbol rate for a signal transmission without intersymbol interference (ISI) according to the Nyquist criterion. The system bandwidth can be limited by a digital narrow-band filter or the analog components. The FTN signaling introduces destructive interference to the higher-order QAM signal, thus more effective equalization schemes are required. In this case, maximum likelihood sequence estimation (MLSE) is widely used at the expense of a relatively high computational complexity [7]–[9]. Besides, a decision feedback equalizer (DFE) provides a low-complexity option [10], which inevitably suffers from the error propagation (EP) effect. By placing the feedback filter at the transmitter, Tomlinson–Harashima precoding (THP) technique can avoid the EP effect [11], [12]. The THP method has been applied with pulse amplitude modulation (PAM) signals showing excellent performances [6], [13]–[20]. For high-capacity IM-DD systems with distances of several tens of kilometers [18], [19], the THP technique can mitigate the influences of both bandwidth narrowing and power fading [2], [20]. Furthermore, the THP was numerically investigated in the coherent system [21], and the FTN THP-QAM signal transmission was studied by simulations [22], [23], and experiment [24], respectively. In the previous THP-based experiments, the THP coefficients were experimentally estimated at the receiver and then applied to the transmitter, resulting in a complex system operation.

In this article, we propose a THP scheme with the numerically estimated coefficients for the FTN QAM signal transmission. The basic principle to numerically estimate THP coefficients is based on the fact that the ISI induced by a digital FTN filter is known at the transmitter [25], [26]. Therefore, we can calculate the THP coefficients to mitigate the FTN-induced ISI, while a receiver-side adaptive equalizer is used to alleviate the ISI caused by the analog components. Since a real-valued baseband simulation is required for the proposed scheme, it excludes the experimental estimation of the THP coefficients, thus enabling a simplified system operation. Simulations are conducted to investigate the tolerances of the proposed scheme to the FTN filtering and channel bandwidth limitation. Furthermore, we perform a proof-of-concept experiment to transmit a 28-Gbaud...
FTN 16-QAM signal over an 80-km single mode fiber (SMF) in an SSBCD system. The results show that the proposed THP scheme can achieve a similar performance as the conventional method, which possesses the same equalization structure as that of the proposed scheme but uses the experimentally estimated THP coefficients. The experimental estimation of the THP coefficients in the conventional scheme is performed by applying an EP-free multiple input multiple output (MIMO) - feedforward equalizer (FFE) - DFE at the receiver, while the transmitter-side THP is excluded [24]. The THP scheme enables a 4.30-b/s/Hz spectral efficiency under a 21.73% FTN rate with a single-polarized 16-QAM signal.

The rest of this article is organized as follows. Section II presents the operation principle of the THP scheme with computed coefficients. Section III shows the numerical verification of the proposed THP scheme based on a coefficient-computation structure. Section IV details the experimental setup as well as the digital signal processing (DSP) algorithms. Section V provides the experimental results and discussion. Finally, Section VI concludes this work.

### II. Operation Principle

When implementing the THP scheme to mitigate ISI, the coefficients of the THP precoder should be first estimated according to the system response. One method of estimating coefficients is to apply an EP-free DFE at the receiver and use the converged DFE coefficients as the THP coefficients [13], [17], [24]. Alternatively, the THP coefficients can be obtained by measuring the frequency response at the receiver and optimizing the tap weights in the frequency domain [6], [14], [20]. These schemes are all performed by experimental means. Fig. 1(a) schematically shows the conventional THP scheme in an FTN system with SSBCD [24]. The THP coefficients are experimentally estimated using an EP-free DFE and then applied to the transmitter for practical THP transmission. To simplify the system operation, we compute the THP coefficients from the transmitter-side FTN filter as shown in Fig. 1(b). The ISI induced by the analog components can be mitigated by the receiver-side FFE. The coefficient-computation structure is depicted in Fig. 1(c). Here, we assume an additive white Gaussian noise (AWGN) channel with the FTN-induced ISI. The receiver consists of an FFE and an EP-free DFE. The coefficients of the FFE and the DFE are updated simultaneously based on the training sequence (TS). Then, the converged DFE coefficients are used as the THP coefficients, which can be recorded in a look-up table (LUT) for practical THP transmission. By excluding the experimental estimation of the THP coefficients, the proposed scheme shows a simplified system operation. Note that a real-valued baseband simulation is performed in the coefficient-computation structure, which possesses a lower computational complexity than the experimental method with complex-valued signal processing. In addition, many DSP blocks are excluded such as field reconstruction and carrier recovery as depicted in Fig. 1(c). The influence of the signal to noise ratio (SNR) in the coefficient-computation structure will be studied in the following section.

### III. Numerical Validation

Numerical simulations are performed to verify the feasibility of the proposed THP scheme. In the presence of noise, the impulse response of the digital FTN filter cannot be used as the THP coefficients, which should be designed to minimize the mean squared error (MSE) of the recovered signal [20]. We compute the THP coefficients based on the transmission model in Fig. 2(a). Since the digital FTN filter is real-valued, we consider a PAM signal in the coefficient-computation structure. A 28-GBaud PAM4 signal is filtered by a digital raised cosine filter (RCF) H(z) for the FTN signaling. The frequency response of the FTN RCF is given in the inset (i), where the bandwidth is 11.5 GHz and the roll-off factor is 0.01. After passing through an AWGN channel with an SNR value of \( SNR_{predicted} \), the FTN PAM4 signal is equalized by a real-valued FFE-DFE. The DFE operates under the EP-free condition, which is realized by feeding the original symbols, rather than the decision values, into the feedback filter. Note that the EP-free mode can only be achieved based on the known TS, which is the original PAM4 symbols in Fig. 2(a). The real-valued tap weights of the feedforward filter F(z) and the feedback filter 1 – B(z) are updated simultaneously using the TS-based least mean square (LMS) algorithm. Therefore, the estimated THP coefficients 1 – B(z) would slightly change at different SNR\(_{predicted}\) values.

By setting the SNR\(_{predicted}\) value to be 21 dB, we obtain the THP coefficients of 1 – B(z). Then, a THP-based FTN QAM signal transmission simulation is conducted in Fig. 2(b). We consider the QAM signal as two independent PAM signals. Thus, the real and imaginary parts of a 28-GBaud 16-QAM signal are processed by two feedback filters with the same coefficients of 1 – B(z), respectively. To limit the output amplitude of each feedback filter, a 2M modulo device adds an integer multiple of 2M to the input signal, thus constraining the pre-coded data to a range of \((-M, M]\). The value of M for the 16-QAM format is 4, since each quadrature component can be regarded as a 4-level signal. The two pre-coded signals are combined and then
filtered by the same FTN RCF as that in Fig. 2(a). The bandwidth limitation from the analog components is emulated by an 8-GHz fifth-order Bessel low-pass filter $C(z)$, and an AWGN is loaded with an SNR of $SNR_{practical}$. Inset (ii) shows the received signal spectrum after normalization, where the $SNR_{practical}$ is 21 dB. For an FTN channel with both the pre- and post-cursor ISIs, a linear equalizer is required at the receiver to recover the signal [20]. In the absence of $C(z)$, the two quadrature components of the received signal can be equalized by two real-valued linear filters with the same fixed coefficients of $F(z)$ obtained from Fig. 2(a), respectively. However, the analog bandwidth limitation is inevitable for a practical transmission system. Therefore, a TS-based adaptive equalizer should be employed at the receiver, which can help alleviate the influence of the limited analog bandwidth. In addition, the actual signal transmission usually suffers from the distortions of in-phase and quadrature (IQ) imbalance and delay, thus we use an adaptive 2-by-2 MIMO-FFE in the proposed scheme. The MIMO structure is robust to the interference between two orthogonal signals [27], which will be discussed in Section V. As shown in Fig. 2(b), TSs are required to extract the coefficients of the MIMO-FFE. After the MIMO-FFE, a relatively flat spectrum is observed in the inset (iii). For a suboptimum $SNR_{predicted}$ value, the THP coefficients of $1 - B(z)$ would be slightly different from that of the optimum $SNR_{predicted}$ value. The difference in $1 - B(z)$ could be minimized by the receiver-side adaptive equalizer, thus similar BERs are observed with varied $SNR_{predicted}$ values. According to Fig. 3, one can empirically set a suboptimum $SNR_{predicted}$ value, which could achieve almost the optimum performance in a practical transmission system.

By varying the bandwidth of the FTN filter, we investigate the FTN-induced performance degradation for the proposed scheme. Here, the channel bandwidth limitation is excluded, and the value of $SNR_{predicted}$ is set to be the same as that of $SNR_{practical}$. Fig. 4 gives the simulation results, where the theoretical curve for an ISI-free 16-QAM signal is added as a benchmark [29]. The SNR penalties for different FTN rates relative to the theoretical curve at a BER of $3.8 \times 10^{-3}$ are summarized in Table I. In a practical transmission system, the FTN rate can be properly selected to achieve a trade-off between the performance and the spectral efficiency.

Then, we investigate the impact of limiting the bandwidth of channel $C(z)$ when using either the conventional way to estimate
the THP coefficients at the receiver or the proposed technique. Note that the THP coefficients in the proposed scheme are computed to alleviate the effect induced by \( H(z) \) in the absence of \( C(z) \). System BER performances of the two schemes, for varying bandwidth of \( C(z) \), are presented in Fig. 5. Both THP schemes use the same transmission mode in Fig. 2(b). The proposed scheme uses the THP coefficients obtained by simulations, while the THP coefficients of the conventional scheme are estimated using a receiver-side MIMO-FFE-DFE detailed in [24]. When implementing the proposed scheme, the \( SNR_{\text{predicted}} \) value is set to be equal to the \( SNR_{\text{practical}} \) value. Compared with the conventional scheme, the proposed scheme causes \( SNR \) penalties of 0.17, 0.62, and 1.35 dB at a BER of \( 3.8 \times 10^{-3} \) for channel \( C(z) \) of bandwidths 10, 8, and 6 GHz, respectively. These \( SNR \) penalties can be attributed to the noise enhancement effect induced by the MIMO-FFE in the proposed scheme. Before the MIMO-FFE, a white noise possesses a flat power spectrum. Then, we plot the noise spectra after the MIMO-FFEs for the two THP schemes with different channel bandwidths in the insets (i-iii). It can be observed that the noise power spectral density is more enhanced at higher frequencies by the MIMO-FFE when using the proposed THP scheme. For the proposed scheme, the frequency response of the MIMO-FFE could possess an inverse amplitude profile of the channel response to mitigate the ISI induced by the channel. Reducing the bandwidth of \( C(z) \) leads to a stronger noise enhancement. However, little noise enhancement is induced by the MIMO-FFE in the conventional case. This is due to the fact that the frequency response of the MIMO-FFE in principle has a constant amplitude for the conventional scheme. As derived in [20], the receiver-side linear equalizer could approximate an all-pass filter. Although the impulse response of the FTN system has both pre-cursor and post-cursor ISIs, the combined impulse response of the FTN system and the receiver-side linear equalizer contains only post-cursor ISI [30]. The resulting post-cursor ISI is pre-mitigated by the THP precoder. By this means, the FTN-related ISI consisting of both pre- and post-cursor ISIs can be suppressed. For the proposed scheme, the THP coefficients are computed to mitigate the influence of \( H(z) \), while \( C(z) \) is excluded. However, the conventional scheme to obtain the THP coefficients at the receiver accounts for both \( H(z) \) and \( C(z) \). Thus, the receiver-side MIMO-FFE of the conventional scheme has less spectral re-shaping to do since parts of \( C(z) \) are already pre-mitigated at the transmitter, which is not the case for the proposed scheme. Therefore, the penalty induced by the proposed scheme increases with a lower bandwidth of \( C(z) \). The penalty related to the noise enhancement could be reduced using a noise whitening filter with an increased complexity [31].

### IV. Experimental Setup and DSP Algorithms

We perform an experiment to investigate the performance of the FTN 16-QAM signal based on the proposed THP scheme. Fig. 6 illustrates the experimental setup of an SSB SCD system [27]. A continuous wave (CW) light from a 15-KHz-linewidth external cavity laser (ECL) at 1550.07 nm is injected into an IQ modulator (IQM) biased at the transmission null. A 64-GSa/s
arbitrary waveform generator (AWG) (Keysight M8195A) generates a 28-GBaud FTN 16-QAM signal, which is amplified by two electrical amplifiers (EAs) and then drives the IQM. The optical 16-QAM signal is combined with a CW tone from another ECL at 1550.165 nm to form an optical SSB signal. The spacing between the two lasers is \(\sim11.88\) GHz. A polarization controller (PC) and a variable optical attenuator (VOA) are employed to align the polarization states and adjust the carrier-to-signal power ratio (CSPR), respectively. The CSPR is defined as the ratio of the CW tone power to the optical 16-QAM signal power, which can be measured after the VOA and the IQM, respectively. The optical SSB signal is boosted by an erbium-doped fiber amplifier (EDFA), and then launched into an 80-km SMF. At the receiver, a VOA is inserted before another EDFA to change the received optical power (ROP), which is measured by a power meter (PM). An optical bandpass filter (OBPF) (EXFO XTM-50) removes the out-of-band noise. After a 50-GHz photodetector (PD), the electrical signal is sampled by a digital storage oscilloscope (DSO) (LeCroy 36Zi-A) at 80 GSa/s.

The transceiver DSP flow charts are shown in Fig. 7. At the transmitter, we generate a signal frame in which the numbers of symbols for synchronization, training, and payload, are 1024, 5120, and 204800, respectively. Then, the THP precoding is performed based on the computed THP coefficients. The precoded signal frame is up-sampled by 2 times and filtered through a 11.5-GHz FTN RCF with a roll-off factor of 0.01. Finally, the digital signal is resampled, clipped, and sent to the AWG. In the receiver DSP, the signal is resampled to a sampling rate of 4 samples per symbol (SPS), followed by the Kramers-Kronig (KK) algorithm to reconstruct the optical field [32]. After chromatic dispersion (CD) compensation, frequency offset compensation (FOC), and synchronization, the MIMO-FFE recovers the signal with an expanded constellation. It should be noted that the coefficients of the MIMO-FFE are extracted based on the extended TSs, which are generated from the original TSs as described in Section III. That means the \(\text{SNR}_{\text{predicted}}\) value on the practical transmission performance. For each \(\text{SNR}_{\text{predicted}}\) value, 20 real-valued THP coefficients are generated. We sweep the \(\text{SNR}_{\text{predicted}}\) values in the numerical simulations to obtain several groups of the THP coefficients, which are recorded in a LUT at the transmitter. Based on these coefficients, we perform BER measurements in the optical back-to-back (OBTB) case and after the 80-km SMF transmission in Fig. 9. Similar to the simulation results, the BER performance shows an independence on the \(\text{SNR}_{\text{predicted}}\) Value, as discussed in Section III. That means the \(\text{SNR}_{\text{predicted}}\) value can be empirically set with a slight performance penalty.

We then evaluate the back-to-back performance of the 28-GBaud FTN 16-QAM signal with different equalization schemes in Fig. 10. In the experiment, the IQ crosstalk originates from IQ amplitude imbalance, IQ phase imbalance, and IQ delay. These distortions could be introduced due to the imperfect components in an optical transmitter. A complex-valued FFE is a complex-valued finite impulse response (FIR) filter, which
cannot mitigate the influence of the IQ crosstalk. It is noteworthy to mention that a real-value 2-input-2-output FFE having 4 trainable real-valued filters applied to the real and imaginary parts of the input can suppress the effect induced by such IQ crosstalk. The 2-by-2 MIMO-FFE is implemented without the transmitter-side THP processing. Similarly, a MIMO-FFE-DFE in [24] is used, which consists of a 2-by-2 MIMO-FFE followed by two real-valued DFEs. The MIMO-FFE-DFE is applied in the absence of the transmitter-side THP. For the conventional THP-MIMO-FFE scheme, a receiver-side EP-free MIMO-FFE-DFE is experimentally applied to estimate the THP coefficients, which are fed back to the transmitter based on a personal computer similar to that in [14]. The details of the conventional THP-MIMO-FFE scheme can be found in [24]. The proposed THP-MIMO-FFE uses the numerically estimated THP coefficients. As shown in Fig. 10, the linear MIMO-FFE does not work in the FTN case. After introducing feedback filters, the MIMO-FFE-DFE achieves a better BER performance. However, the inevitable EP effect leads to a high error floor. By using the THP-MIMO-FFE schemes, the EPs can be avoided, thus the BERs can be significantly reduced. Comparing the two THP-MIMO-FFE schemes, it is observed that the proposed scheme has a sensitivity penalty of \( \sim 0.6 \text{ dB} \). The small penalty in the experiment can be explained as such: the system bandwidth is relatively large, and the analog channel does not truncate the signal bandwidth below 11.5 GHz. With the similar performance in the conventional scheme, the proposed scheme can simplify the system operation without the need for experimentally estimating THP coefficients.

For the 80-km SMF transmission, the launch power and the CSPR should be optimized. As presented in Fig. 11, the minimum BER value is achieved with a 13-dB CSPR and a \( \sim 5\text{-dBm} \) launch power, which are used in the following BER measurements. Fig. 12 gives the BER curves for different equalization schemes after the fiber transmission. Similarly, the THP-MIMO-FFE scheme outperforms other equalizers, by using numerically or experimentally estimated THP coefficients. The sensitivity difference between the proposed and the conventional schemes is negligible. Both the two THP schemes can reach BERs lower than the 7% FEC threshold after the 80-km transmission.

With the coefficients of \( 1 - B(z) \), a THP precoder possesses a response of \( 1/B(z) \). In Fig. 13, we plot several groups of \( B(z) \) estimated by numerical and experimental methods, respectively. Both impulse and frequency responses of \( B(z) \) are provided to show the influence of the channel bandwidth limitation. The frequency response is obtained by padding 491 zeros for each \( B(z) \) to perform a 512-point fast Fourier transform (FFT). For the proposed scheme, 21- and 19-dB SNR\text{predicted} values are
Fig. 12. BER curves for different equalization schemes after the 80-km transmission. (i-ii) Constellation diagrams at the –11-dBm ROP for the THP-free MIMO-FFE and the THP-free MIMO-FFE-DFE, respectively. (iii-iv) Constellation diagrams at the –11-dBm ROP for the proposed THP-MIMO-FFE scheme before and after the modulo operations, respectively.

Fig. 13. Impulse and frequency responses of $B(z)$ at different cases.

Fig. 14. BER curves for the proposed THP scheme with the MIMO-FFE and the complex-valued FFE, respectively.

**VI. CONCLUSION**

We have proposed and demonstrated an FTN QAM signal transmission system based on the THP scheme with the numerically estimated coefficients. In our scheme, a real-valued baseband simulation is performed considering a digital FTN filter.
in an AWGN channel. The resulting THP coefficients are then applied to the transmitter for practical transmission. Compared with the experimental estimation of the THP coefficients, the numerical method can simplify the system operation. We perform simulations to investigate the performance of the proposed scheme in the presence of FTN signaling and channel bandwidth limitation. A proof-of-concept experiment is conducted to transmit a 28-Gbaud FTN 16-QAM signal over an 80-km SMF in an SSB SCD system. The proposed scheme causes a negligible sensitivity penalty relative to the conventional method in the experiment. A record 21.73% FTN rate with a high spectral efficiency of 4.30 b/s/Hz is achieved. The experimental results show that the proposed scheme could be a promising solution to realize the spectrally efficient DD system with the FTN QAM format. In addition, the proposed scheme can in principle be applied to a coherent dense WDM system to further improve the spectral efficiency.

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