Abstract—Neuromorphic photonics that aims to process and store information simultaneously like human brains has emerged as a promising alternative for next generation intelligent computing systems. The implementation of hardware emulating the basic functionality of neurons and synapses is the fundamental work in this field. However, previously proposed optical neurons implemented with SOA-MZIs, modulators, lasers or phase change materials are all dependent on active devices and pose challenges for integration. Meanwhile, although the nonlinearity in nanocavities has long been of interest, the previous theories are intended for specific situations, e.g., self-pulsation in microrings, and there is still a lack of systematic studies in the excitability behavior of the nanocavities including the silicon photonic crystal cavities. Here, we report for the first time a universal coupled mode theory model for all side-coupled passive microresonators. The excitability of microresonators resulting from the subcritical Andronov-Hopf bifurcation is categorized as Class 2 neural excitability and can be used to mimic the Hodgkin-Huxley neuron model. We demonstrate that the microresonator-based neuron can exhibit the typical characteristics of spiking neurons: excitability threshold, leaky integrating dynamics, refractory period, cascadability and inhibitory spiking behavior, paving the way to realize all-optical spiking neural networks.

Index Terms—Coupled mode theory, microresonators, nonlinearity, neuromorphic photonics, optical neurons.

I. INTRODUCTION

Artificial intelligence (AI) has attracted a lot of attention lately because of its great potential to remarkably change almost every aspect of our daily life [1]. Applications of AI such as face recognition [2], intelligent virtual assistant [3] and neural machine translation [4] have already been widely used and brought great convenience to us. However, it takes a considerable amount of computational time to train the neural network for a specific task and AI operating on traditional computers consumes great power as well. In the von Neumann computing architecture, the computing unit and the memory are physically separated from each other, thus the machine instructions and data are serially transmitted between them. Consequently, the computational capacity is limited by the bandwidth and high energy consumption. Compared to the massively parallel processing of human brains, it’s undoubtedly inefficient to train and simulate a neural network with software on a von Neumann computer.

Neuromorphic computing takes inspiration from human brains and aims to simultaneously process and store information as fast as possible, which provides a promising alternative for next generation intelligent computing systems. It develops hardware mimicking the basic functions of neurons and synapses, and then combines them into suitably scaled neural networks. In the past few years, neuromorphic electronics, e.g., TrueNorth at IBM [5], TPU at Google [6] and SpiNNaker at the University of Manchester [7], have demonstrated impressive improvements in both energy efficiency and speed enhancement, allowing simultaneous operation of thousands of interconnected artificial neurons. However, electronic architectures face fundamental limits as Moore’s law is slowing down [8]. Furthermore, the electronic communication links have intrinsic limitations on speed, bandwidth and crosstalk. Distinct from those properties of electronics, silicon photonics naturally has the advantages of low latency and low power consumption. Data is transmitted in silicon waveguides at the speed of light and the associated energy costs currently are just on the order of femtoujoules per bit [9]. Combined with the dense wavelength-division multiplexing (WDM), mode division multiplexing (MDM) and polarization division multiplexing (PDM) technologies, the channel capacity continues to increase. Consequently, neuromorphic photonic systems could operate 6–8 orders-of-magnitude faster than neuromorphic electronics [10].

Synapses and neurons are the basic building blocks of human brains, which perform the linear weight-and-sum operation and the nonlinear activation operation. To obtain a complicated brain-inspired optical chip, the hardware implementation of the fundamental function units is essential. Recently, a number of different concepts for neuromorphic computing have been proposed, including the reservoir computing [11]–[17], artificial neural networks (ANN) [18]–[25] and spiking neural networks (SNN) [26]–[44]. Utilizing the linear optics offered by the native interference properties of light makes it possible to implement...
In this paper, we first propose, to the best of our knowledge, a universal CMT model for all passive side-coupled microresonators. The self-pulsation and excitability behaviors are successfully simulated to validate the presented model. We point out that the excitability of passive microresonators can be categorized as Class 2 neural excitability and used to mimic the Hodgkin-Huxley neuron model. Then, we demonstrate that the excitability of passive microresonators can be described as follows [48]:

\[
\frac{da_{\pm}}{dt} = \left[ j(\omega' - \omega_{\pm}) - \frac{1}{2\tau_{\text{total}} S_{\pm}} \right] a_{\pm} + \kappa_{\pm} S_{\pm},
\]

(1)

\[
\frac{dN}{dt} = -\frac{N}{\tau_{fe}} + \frac{\Gamma_{\text{FCA}} \beta_{\text{Si}} \epsilon_0^2}{2h\omega V_{\text{FCA}} A_g^2} \left( |a_+|^4 + 4|a_+|^2 |a_-|^2 + |a_-|^4 \right).
\]

(2)

\[
\frac{d\Delta T}{dt} = -\frac{\Delta T}{\tau_{th}} + \frac{\Gamma_{\text{th}} P_{\text{abs}}}{\rho_S C_p Si V_{\text{cavity}}}.\]

(3)

II. COUPLED MODE THEORY OF MICRORESONATOR

The nonlinearity of microresonators has long been of interest as a simple mechanism to manipulate light with light. The nonlinear phenomena have already been widely observed and carefully studied in different material platforms and various types of integrated resonators. All the self-pulsation, bistability and excitability behavior have been experimentally demonstrated in microrings [45]–[49], microdisks [50], [51] and two-dimensional photonic crystal (PhC) cavities in InP platform [52], [53]. The silicon two-dimensional PhC cavities can also exhibit bistability [54]–[56] and self-pulsation [57]–[60] behaviors. Besides, bistability has been observed in racetrack resonator [61] and silicon one-dimensional PhC cavity [62], [63]. While excitability behavior hasn’t been demonstrated in silicon PhC nanocavities.

The basic principle of nonlinear behaviors for Silicon-On-Insulator (SOI) microresonators is [48]: two-photon absorption (TPA) effect first generates free carriers, which are then able to absorb light by free carrier absorption (FCA) effect. Besides, the presence of free carriers leads to a blueshift in the resonance wavelength by free carrier dispersion (FCD) effect. Moreover, surface state absorption takes place everywhere at the silicon-silica interface, and meanwhile some light is lost due to the surface scattering and radiation loss. Therefore, the absorbed optical energy is mainly lost in the form of heat. Owing to the thermo-optic effect, the heat induces a redshift in the resonance wavelength. Typically, the free carriers relax at least one order of magnitude faster than the heat. Consequently, this difference between the timescale of the fast free carrier dynamics and the slow heating effects results in the self-pulsation, bistability and excitability phenomena in microresonators.

Previously, different forms of CMT have been proposed to analyze the nonlinearity in silicon microrings [46]–[48], microdisks [50], [51] and two-dimensional silicon PhC cavities [57], [59], [60], [64]. However, these CMT equations are intended for specific situations, e.g., self-pulsation in microrings. The lack of systematic studies in the nonlinear excitability behavior of nanocavities including the silicon PhC cavities is pressing.

In this paper, we first propose, to the best of our knowledge, a universal CMT model for all passive side-coupled microresonators. Considering the most common situation where a passive microresonator couples with only one waveguide, the nonlinear dynamics of the microresonator can be described as follows [48], [50], [55], [65], [66]:
where \( a_\pm \) is the complex amplitude of the forward and backward propagation mode respectively and \( |a_\pm|^2 \) stands for the corresponding mode energy in the microresonator. \( S_\pm \) represents the complex amplitude of the pump light and the perturbation signal injected in the opposite direction. \( \omega_\pm = \frac{2n_\pm}{\lambda} \) is the frequency of input light in the waveguide, \( \omega' = \omega_r + \Delta\omega_i \) denotes the shifted resonance frequency of the microresonator, where \( \omega_r = \frac{2n_r}{\lambda} \) is the original resonance frequency and \( \Delta\omega_i \) is the total frequency shift caused by all nonlinear effects. \( \kappa_\pm = \sqrt{\frac{1}{\tau_{in,\pm}^\text{tau}}} \) is the coupling coefficient between the waveguide and the microresonator. \( N \) is the concentration of free carriers in the microresonator and \( \Delta T \) is the mode-averaged temperature difference with the surroundings. \( \tau_{th} \) is the relaxation time for temperature and \( \tau_{fc} \) is the effective free-carrier delay rate accounting for both recombination and diffusion. \( \beta_Si \) is the constant governing TPA, \( \rho_{Si} \) is the density of silicon, \( C_{p, Si} \) is the thermal capacity of silicon and \( V_{cavity} \) is the volume of the microresonator. \( n_g \) is the group index, generally, the dispersion is neglected and \( n_g = n_{Si} \), \( n_{Si} \) is the refractive index of bulk silicon. \( V_a \) and \( \Gamma_a \) denote the effective mode volume and confinement coefficient.

The total loss for each cavity mode is:

\[
\frac{1}{\tau_{i,\text{total}}} = \frac{1}{\tau_{i,\text{lin}}} + \frac{1}{\tau_{i,v}} + \frac{1}{\tau_{i,\text{TPA}}} + \frac{1}{\tau_{i,\text{FCA}}}.
\]

(4)

where \( \frac{1}{\tau_{i,\text{lin}}} \) is the linear absorption loss, \( \frac{1}{\tau_{i,v/v}} \) represents the in-plane waveguide coupling loss and the vertical radiation loss respectively, given by \( \frac{1}{\tau_{v/v}} = \frac{1}{\tau_{i,\text{TPA}}} + \frac{1}{\tau_{i,\text{FCA}}} \) are the loss due to TPA and FCA respectively. The mode-averaged TPA loss rates are:

\[
\frac{1}{\tau_{i,\text{TPA}}} = \Gamma_{\text{TPA}} \frac{\beta_Si c^2}{n_g^2 V_{TPA}} (|a_\pm|^2 + 2|a_\mp|^2).
\]

(5)

The mode-average FCA loss rate is defined as:

\[
\frac{1}{\tau_{i,\text{FCA}}} = \frac{c}{n_g} (\sigma_e + \sigma_h) N.
\]

(6)

\( \sigma_e/h \) is the absorption cross-section for electrons and holes.

The total detuning of the resonance frequency of the microresonator \( \Delta\omega_i \) can be expressed by:

\[
\frac{\Delta\omega_i}{\omega_r} = \frac{\Delta n_{i,Kerr}}{n_{Si}} = \left( \frac{\Delta n_{i,Kerr}}{n_{Si}} + \frac{\Delta n_{i,FCD}}{n_{Si}} + \frac{\Delta n_{i,th}}{n_{Si}} \right).
\]

(7)

The detuning due to the Kerr effect is:

\[
\frac{\Delta n_{i,Kerr}}{n_{Si}} = \frac{c n_2}{n_g V_{Kerr}} (|a_\pm|^2 + 2|a_\mp|^2).
\]

(8)

where the effective mode volume for Kerr effects \( V_{Kerr} = V_{TPA} \) and \( n_2 \) is the Kerr coefficient. The detuning due to the FCD effect can be expressed by:

\[
\frac{\Delta n_{i,FCD}}{n_{Si}} = -\frac{1}{n_{Si}} \frac{d n_{Si}}{d N} N = -\frac{1}{n_{Si}} (\xi_{e} + \xi_{h}) N.
\]

(9)

Here \( \xi_{e/h} \) is the material parameter. Finally, the detuning due to the thermal dispersion is:

\[
\frac{\Delta n_{i,th}}{n_{Si}} = -\frac{1}{n_{Si}} \frac{d n_{Si}}{d T} \Delta T.
\]

(10)

The total absorbed power is given by:

\[
P_{\text{abs}} = P_{\text{abs,lin}} + P_{\text{abs,TPA}} + P_{\text{abs,FCA}}.
\]

(11)

\[
P_{\text{abs,lin}} = \frac{1}{\tau_{i,\text{lin}}} \left( |a_+|^2 + |a_-|^2 \right).
\]

(12)

\[
P_{\text{abs,FCA}} = \frac{1}{\tau_{i,\text{FCA}}} \left( |a_+|^2 + |a_-|^2 \right).
\]

(13)

\[
P_{\text{abs,TPA}} = \Gamma_{\text{TPA}} \frac{\beta_Si c^2}{n_g^2 V_{TPA}} (|a_+|^4 + 4|a_+|^2|a_-|^2 + |a_-|^4).
\]

(14)

### III. Nonlinear Behaviors of Microresonators

To validate the universal CMT-equations presented above, we simulate the nonlinear self-pulsation and excitability behaviors of microresonators taking the most common microrings and nanobeams as examples, and the parameters used in the simulation are provided in Table I. Since it takes a relatively long time for the microring to reach its steady state from scratch, all the simulation results for microrings begin with the steady state in demonstration. In simulation the pump light and the perturbation signal have the same wavelength, and the backscattering is neglected. As a consequence, there exists no interference effect due to the coupling between the pump light and the trigger light. Moreover, the dynamics of the microresonator will be independent of the phase of the perturbation signal. The time width of the trigger pulse is 10 ns and 1 ns for the microring and the nanobeam respectively. Besides, the maximum power of the perturbation signal is set to be the corresponding pump power.

The optical energy in the microresonator generates both heat and free carriers, while the thermo-optic effect and the FCD effect have an opposite influence on the resonance frequency shift. Due to the difference in the timescale between the slow relaxation process of heat and the fast free carrier generation and absorption dynamics, microresonators will exhibit self-pulsation behavior, as shown in Fig. 2. The mode energy in the microcavity, the temperature difference between the microcavity and the surroundings, the concentration of free carriers, and the power of output light are all evolving periodically with time.

In fact, if the input power is below but very close to the self-pulsation power, the microresonator can be excited to emit a “negative” output pulse when applied a strong enough perturbation signal. As shown in Fig. 3, the microresonator initially rests at its steady state and the output power remains constant. When triggered with a strong enough perturbation signal, the microresonator will be kicked out from its equilibrium. All the mode-averaged cavity energy, the temperature difference between the resonators and their surroundings and the concentration of free carriers go through a sudden increase and then slowly recover to their steady states, resulting in the appearance of a “negative” pulse in output power. Moreover, as long as the power of the trigger signal is above the threshold value, the shape of the “negative” output pulse will keep the same. The
TABLE I
PARAMETERS FOR MICRORING AND NANOBRE [48], [55], [65], [66]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Microring</th>
<th>Nanobeam</th>
<th>Units</th>
</tr>
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<td>$\lambda_r$</td>
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<td>1552.45</td>
<td>nm</td>
</tr>
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<td>$Q$</td>
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<tr>
<td>$Q_{in}$</td>
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<td>$Q_{in}$</td>
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<td>ns</td>
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<td>10$^{-21}$</td>
<td>m$^3$</td>
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<td>m$^3$/W</td>
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<td>$e$</td>
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<td>1.6×10$^{-19}$</td>
<td>C</td>
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<td>$\varepsilon_0$</td>
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<td>0.3×$m_0$</td>
<td>kg</td>
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<tr>
<td>$m_h$</td>
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<td>kg</td>
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<td>$dn_{Sl}/dT$</td>
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<td>1.86×10$^{-4}$</td>
<td>$K^{-1}$</td>
</tr>
</tbody>
</table>

Simulation results agree quite well with previous studies [48], [49], which guarantees the correctness of the proposed universal CMT-equations.

IV. OPTICAL SPIKING NEURON BASED ON PASSIVE MICRORESONATOR

Although the microresonator emits a “negative” pulse when excited with a perturbation signal, it can still be used to mimic the basic functions of a spiking neuron. In SNN, information is encoded and transmitted in the form of spikes. It doesn’t matter what the shape of the spike is, and whether the spike is “positive” or “negative” also makes no difference to the operation of the whole SNN. Here, we explicitly point out that the passive microresonator can serve as an all-optical spiking neuron. According to the phase-plane analysis [48], the excitability of passive microrings can be explained by the subcritical Andronov-Hopf bifurcation when the input light is blue detuned, which results in Class 2 neural excitability. We believe the proposed neuron can mimic the Hodgkin-Huxley neuron model as the Andronov-Hopf bifurcations have been scrutinized numerically and analytically in this model [67]–[72]. Therefore, the microresonator-based neuron should be categorized as Class 2 neuron.

The spiking neuron has several typical characteristics: (i) excitability threshold: The spiking neuron will only emit an output pulse when the power of the perturbation signal exceeds a certain threshold. Otherwise, the neuron will not respond and remain silent. (ii) leaky integrating dynamics: The spiking neuron can be excited with several sufficiently closely-spaced subthreshold pulses by integrating their combined perturbation (iii) refractory period: After an excitation, the neuron needs to relax to its steady-state before it can be triggered once again, and the recovery time needed is called the refractory period. (iv) cascadability: The output spike of the previous neuron should be strong enough to excite the next neuron, which is the foundation of forming a multi-layer SNN. (v) inhibitory spiking behavior: The arrival of a stimulus can stop the spiking activity, which plays an important role in the well-known spike-timing-dependent plasticity (STDP) learning rules [73].

Fig. 2. Schematics of the self-pulsation behavior of microresonators. (a) For microrings: $\delta_1 = -20$ pm and $P_{in} = 5$ mW; (b) For nanobeams: $\delta_1 = -12$ pm and $P_{in} = 5.6$ $\mu$W.
Next, we provide a detailed exposition of these characteristics for the nanobeam-based spiking neuron. The microring-based neuron can exhibit the same behaviors as nanobeams, and the simulation results are available in [74].

A. Excitability Threshold

In ANN, the nonlinear unit is a necessary part for learning tough tasks. Without the nonlinear activation function, the output of ANN will only be the simple linear combination of the input information. Consequently, the ANN turns out to be the primitive perceptron machine. It’s the nonlinear unit that makes it possible for ANNs to approximate any functions, thus realizing complicated tasks such as image classification, speech recognition and so on. The sigmoid function and tanh function are usually adopted in the ANN. In contrast, the information in SNN is encoded in the timing or rate information of spikes, which are assumed to be all-or-none binary events. The spikes are used for interneuron communication and carry information through the sheer fact of their appearance, whereas their amplitude and width are neglected. Consequently, the condition on which the spiking neuron can be triggered to emit an output spike, or more exactly, the excitability threshold behavior, directly determines the performance of the SNN.

In the case of the subcritical Andronov-Hopf bifurcation, there doesn’t exist a well-defined threshold manifold, and a continuous transition between subthreshold oscillations and excitability usually exists [75]. We define the normalized strength of the “negative” output pulses as: $S = 1 - \frac{P_{\text{min}}}{P_{\text{constant}}}$, where $P_{\text{min}}$ and $P_{\text{constant}}$ represent the minimum of the output power after the perturbation pulses and the constant output power at the steady state respectively. To determine the threshold of our proposed neuron under specific pump conditions, we fix the duration $T_{tr}$ and change the power $P_{tr}$ of the perturbation pulse, and we define the threshold value at the point where the slope $dS/dP_{tr}$ is the maximum. For $\delta \lambda = -12 \text{ pm}$, $P_{in} = 4.4 \text{ mW}$, $T_{tr} = 1 \text{ ns}$, Fig. 4 presents how the normalized strength changes with the power of one single perturbation pulse. The normalized strength first increases slowly when $P_{tr}$ is below $1.25 \text{ mW}$, then it experiences a sharp jump when $P_{tr}$ increases from $1.25 \text{ mW}$ to $1.4 \text{ mW}$. After that, the normalized strength increases very slowly again and remains almost constant when $P_{tr}$ is above $1.5 \text{ mW}$. $P_{tr} = 1.36 \text{ mW}$ is defined as the threshold where the slope of the curve is the maximum at this point. With one single trigger pulse, perturbation above the threshold will result in a remarkable stronger strength increase than the subthreshold perturbation and the spiking shape will keep almost constant. However, no functional excitability will be induced in the low-$P_{tr}$ region of one single trigger, as the shape of the “negative” output pulse is dependent on the perturbation power and the normalized strength isn’t strong enough.

Fig. 5 presents the distribution of the excitability threshold of nanobeams over different input power and resonance shift. It’s worth mentioning that the microresonator will exhibit self-pulsation or stay at the steady state at the upper right gray region. Moreover, it can be excited at the lower left gray region, but the trigger power needs to be higher than the pump power, which is not discussed in this paper. According to Fig. 5, as the wavelength shift increases, the excitability threshold power of microresonators increases gradually. On the contrary, it becomes...
lower when pumped with higher input power. In addition, the excitability threshold is more sensitive to the wavelength shift than the pump power.

B. Leaky Integrating Dynamics

Similar to the leaky integrate-and-fire (LIF) neuron, the nanobeam-based neuron can also be excited by integrating several closely-spaced subthreshold perturbation pulses. To demonstrate this, we keep $\delta \lambda = -12$ pm, $P_{in} = 4.4$ $\mu$W, $P_{tr} = 1$ $\mu$W and $T_{tr} = 1$ ns. As shown in Fig. 6, when a perturbation pulse is sent to the nanobeam at $t = 20$ ns, small variations are introduced to the mode energy, the temperature shift and the concentration of free carriers in the nanobeam. Consequently, the output power experiences slight attenuation and then recovers slowly to the steady state. At $t = 40$ ns, two identical perturbation pulses with an interspike interval of 1 ns are applied to the nanobeam. As opposed to the case of single pulse, the output power attenuates to almost 0 $\mu$W and a distinct “negative” spiking dip output pulse is generated. The nanobeam-based neuron can also be excited by two subthreshold perturbation pulses, which is the direct evidence of the integrating behavior. Moreover, during the time interval from $t = 41$ ns to $t = 42$ ns, the mode energy, the temperature variation and the concentration of free carriers decrease slowly, and the output spiking dip experiences a weak recovery approximately from 2 $\mu$W to 2.2 $\mu$W, which means the integrating process is leaky. We also notice that the two same perturbation pulses have different impacts on the nanobeam, which can be attributed to the resonance wavelength shift caused by the first perturbation pulse. For example, the first perturbation pulse leads to the output power attenuates from about 3 $\mu$W to 2 $\mu$W, while the second one causes the output power attenuates from about 2.2 $\mu$W to 0 $\mu$W.

To further study the timing information during this process, we change the interspike interval between the two identical subthreshold perturbation pulses and record the normalized strength. For $\delta \lambda = -12$ pm, $P_{in} = 4.4$ $\mu$W, $P_{tr} = 1$ $\mu$W and $T_{tr} = 1$ ns, the response of the nanobeam to two identical perturbation pulses is depicted in Fig. 7. The normalized strength remains almost constant when the interspike interval is below 1 ns and decreases very slowly when the interval increases from 1 ns to 1.45 ns. Actually, almost constant output spiking pulses can be observed in this region. Then the normalized strength decreases sharply from about 0.9 to 0.6 when the interval increases from 1.45 ns to 1.7 ns. After that, the normalized strength
Fig. 8. Schematics of the refractory period behavior of nanobeams. For $\delta \lambda = -13 \text{ pm}$, $P_{in} = 5 \mu W$, $P_{tr} = 2 \mu W$, (a) the nanobeam keeps silent to the second trigger pulse when the time interval is 12 ns, (b) but it can be excited once again when the time interval increases to 16 ns.

strength decreases slowly again and remains about 0.3 when the interval exceeds 2.5 ns.

C. Refractory Period

After an excitation, the spiking neuron will be insensitive to the second trigger signal until recovering to its steady state. When scaled to a multi-layer SNN, the refractory period will directly determine the operation speed of the whole network.

The microresonator-based optical neuron has this property as well. As shown in Fig. 8, if the interval between the two trigger pulses is shorter than the refractory period, the microresonator will not respond to the second perturbation signal. For $\delta \lambda = -13 \text{ pm}$, $P_{in} = 5 \mu W$, $P_{tr} = 2 \mu W$ when the time interval is 12 ns, the nanobeam stays silent to the second trigger pulse. However, when the time interval increases to 16 ns, the nanobeam emits an output spike once again, which is the direct evidence of the refractory period behavior. The effective relaxation time of free carrier $\tau_{fc}$ and the relaxation time of temperature $\tau_{th}$ are two key factors deeply influencing the excitability behavior of microresonators. The former determines the shape of the “negative” pulse, while the latter determines how fast the microresonator can be excited. Beneficial from the shorter thermal relaxation time, the refractory period for nanobeams ($\sim 20 \text{ ns}$) is approximately one eighth of that for microrings ($\sim 160 \text{ ns}$).

D. Cascadability Behavior

To provide the required learning capacity for complicated artificial intelligence applications such as computer vision and medical diagnosis, a multi-layer neural network is the inevitable choice, which means the cascadability behavior of optical neurons is quite necessary. In order to simulate the cascadability behavior of the proposed optical neuron, two microresonators with the same conditions are cascaded. The perturbation signal is only applied to the first microresonator, and the output signal of the first neuron directly serves as the input signal of the second neuron. The simulation results are presented in Fig. 9. For $\delta \lambda = -18.5 \text{ pm}$, $P_{in} = 11 \mu W$, $P_{tr} = 0.75 \mu W$ the perturbation signal excites the first nanobeam and its “negative” output pulse triggers the second nanobeam. All the physical parameters of the second microresonator including the mode-averaged energy in the cavity, the temperature difference with the surroundings and the free carrier concentration evolve in the same way as the first microresonator after a certain latency. Consequently, there exist two “negative” pulses at the output of the second microresonator. The first “negative” pulse corresponds to the output of the first neuron, and the second pulse is the triggered output of the second neuron, which is the direct evidence for the cascadability behavior of the microresonator-based spiking neuron.

E. Inhibitory Spiking Behavior

Both the spiking excitation and inhibition are integral parts of the STDP learning rules. Recently, some innovative schemes have been proposed to experimentally demonstrate the excitatory and inhibitory spiking dynamics simultaneously in neuromorphic photonic systems [40]–[43]. We point out that the proposed neuron can also emulate the inhibitory spiking behavior. We keep $\delta \lambda = -12 \text{ pm}$, $P_{in} = 4.4 \mu W$, $T_{tr} = 1 \text{ ns}$.
Unlike the previous setup, the power of the perturbation light is set to be 0.35 μW at the steady state. Under these conditions, a single excitatory perturbation pattern with the high level of 1 μW can excite the nanobeam. To demonstrate the inhibitory spiking dynamics, inhibitory perturbation patterns with the low level of 0 μW are sent to the nanobeam. As shown in Fig. 10, as the inhibitory pattern moves toward the excitatory pattern, the “negative” output spike is gradually suppressed to almost null; as the inhibitory pattern moves far away from the excitatory pattern, the “negative” output spike recovers to the normal state again. Therefore, the proposed neuron can emulate both the excitatory and inhibitory spiking dynamics.

V. DISCUSSION

We analyze the performance of the microresonator-based neuron: the power consumption and the operation speed are determined by the pump power and the refractory period respectively. As discussed above, the microring-based neuron can be pumped and excited with serval milliwatts. Owing to the ultra-high Q/V ratio, the light-matter interactions in the nanobeam is much stronger than that in the microring. Consequently, the nanobeam-based neuron can work under serval microwatts, three orders-of-magnitude lower than the microring. Moreover, since the refractory period is mainly limited by the relaxation time of heat, the nanobeam-based neuron can operate more than eight times faster than the microring-based neuron. We emphasize that the nanobeam-based neuron can operate at the speed of GHz, six orders-of-magnitude faster than the timescale of biological neural networks: kHz. Table II depicts the performance comparison between our proposed optical neuron with previous work. The microresonator-based neuron can process information passively, which is extremely difficult for other types of neuron to achieve. Moreover, they can be easily fabricated in photonic integrated circuits together with other optical components in standard CMOS process, thus making large-scale SNNs available. Combined with the PCM-implemented synapse in [30], an all-optical passive spiking neural network can be obtained.

Furthermore, we investigate the impact of Q factor on the distribution of the cascadability behavior of the nanobeam-based neuron. Since ultra-high Q of $3.6 \times 10^5$ on SiO2 claddings and $7.2 \times 10^5$ on air claddings have been experimentally demonstrated [76], we keep the $Q_v/Q_{in}$ ratio constant and increase the Q value from original 114400 to 171600 and 228800 successively. The simulation results are depicted in Fig. 11. As the value of Q increases from left to right, relatively larger wavelength detuning and higher pump power are needed for the nanobeam-based neuron to demonstrate the cascadability behavior. It can be attributed to the high temperature sensitivity of the resonance frequency resulting from the ultra-high Q/V ratio.

Finally, we give the general guideline to design a practical microresonator-based neuron: (i) Choose the type of microresonator. Compared with microrings, the nanobeam-based neuron can work with faster speed and much lower power. However, owing to the ultra-high Q/V ratio, nanobeams can be more sensitive to the temperature variation of the surroundings, affecting the stability of the whole system. Moreover, the condition for the cascadability of nanobeams is more critical; (ii) Design the quality factor $Q_v$ and $Q_{in}$ of the microresonator. Generally, a higher Q can enhance the light-matter interaction, thus making the microresonator exhibit the expected nonlinear behaviors. While as discussed above, a lower Q is preferred to more easily achieve the cascadability with lower threshold; (iii) Choose

<table>
<thead>
<tr>
<th>Type</th>
<th>Power</th>
<th>Speed</th>
<th>Cascadability</th>
<th>Integration</th>
<th>Passive/active</th>
</tr>
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<tbody>
<tr>
<td>Microring (proposed)</td>
<td>SNN</td>
<td>mW</td>
<td>sub-μs</td>
<td>Yes</td>
<td>High</td>
</tr>
<tr>
<td>Nanobeam (proposed)</td>
<td>SNN</td>
<td>μW</td>
<td>ns</td>
<td>Yes</td>
<td>high</td>
</tr>
<tr>
<td>SOA-MZI [19,38]</td>
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<td>mW</td>
<td>sub-μs</td>
<td>Yes</td>
<td>Low</td>
</tr>
<tr>
<td>Laser cooled atoms [25]</td>
<td>ANN</td>
<td>sub-μW</td>
<td>ns</td>
<td>Yes</td>
<td>Low</td>
</tr>
<tr>
<td>Modulator [25,24]</td>
<td>ANN</td>
<td>mW</td>
<td>ns</td>
<td>Yes</td>
<td>medium</td>
</tr>
<tr>
<td>Laser [26,27,35-37,40-42]</td>
<td>SNN</td>
<td>μW / mW</td>
<td>sub-μs/μs</td>
<td>Yes</td>
<td>medium</td>
</tr>
<tr>
<td>PCM [29,31]</td>
<td>SNN</td>
<td>mW</td>
<td>ns</td>
<td>No</td>
<td>medium</td>
</tr>
</tbody>
</table>

Fig. 10. Inhibitory spiking dynamics of the proposed neuron. From the left to the right, the excitatory pattern is lagging the inhibitory pattern by 3 ns, 1 ns, and 0 ns for the first three cases, and it is leading by 1.4 ns, 3 ns for the last two cases.
Fig. 11. Schematic of the influence of Q factor on the distribution of the cascadability behavior of nanobeam-based neuron. From left to right, Q = 114400, Q = 171600, and Q = 228800 respectively.

VI. CONCLUSION

In summary, we creatively utilize the excitability of passive microresonators to emulate the functions of spiking neurons. The microresonator-based neuron is categorized as Class 2 neuron and can be used to mimic the Hodgkin-Huxley model. We demonstrate that the proposed neuron can exhibit the excitability threshold, leaky integrating dynamics, refractory period, cascadability and inhibitory spiking behavior, which are essential to implement a feasible hardware SNN. We emphasize that the proposed neuron is completely passive and can be easily fabricated with standard CMOS process. Moreover, we report a universal CMT model that can be used to analyze the nonlinear dynamics in all side-coupled microresonators, especially for the excitability of silicon PhC cavities. Combining the PCM-based synapse and the microresonator-based neuron, multi-layer all-optical spiking neural networks can be implemented.

REFERENCES


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